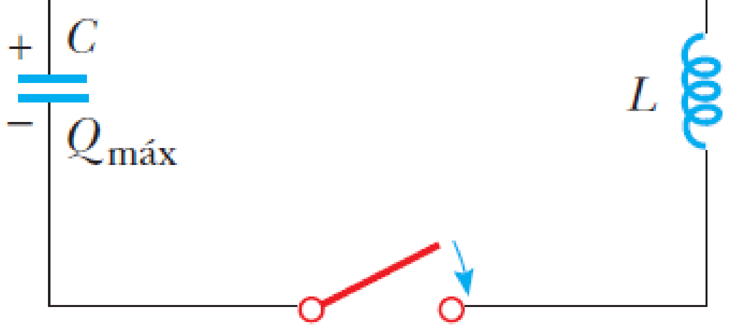
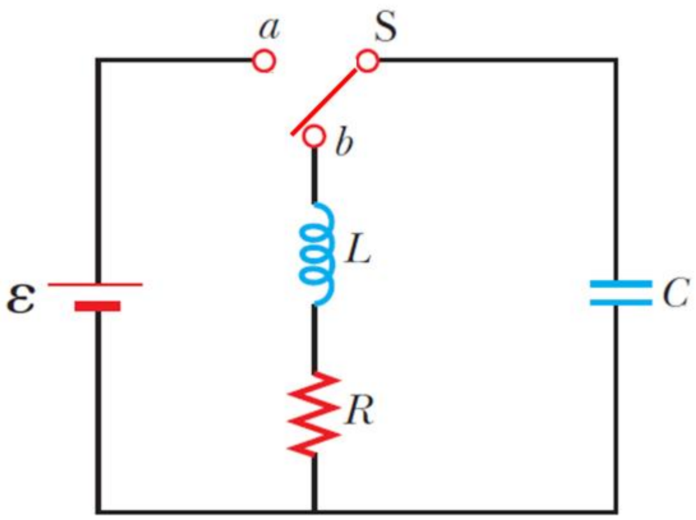




Circuitos Oscilantes

Inducción mutua

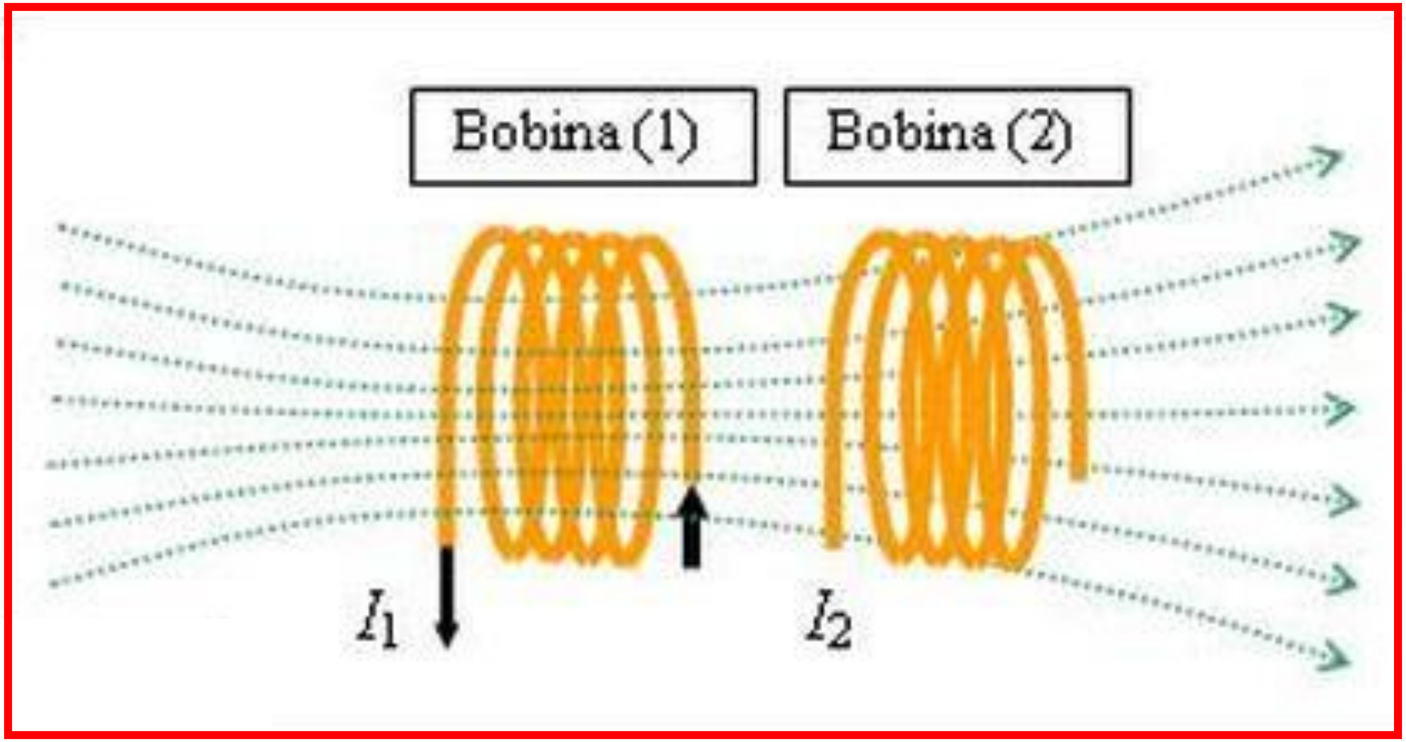
Transformadores

Oscilaciones Libres (sin fuente):

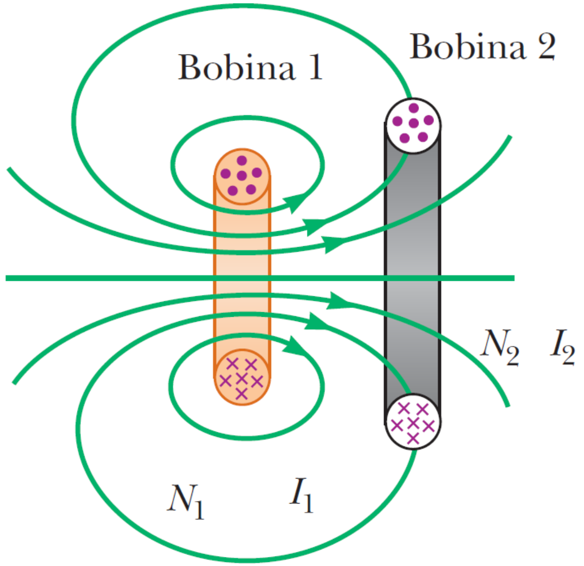
- Circuitos LC

- Circuitos RLC

Inducción mutua



Inducción mutua



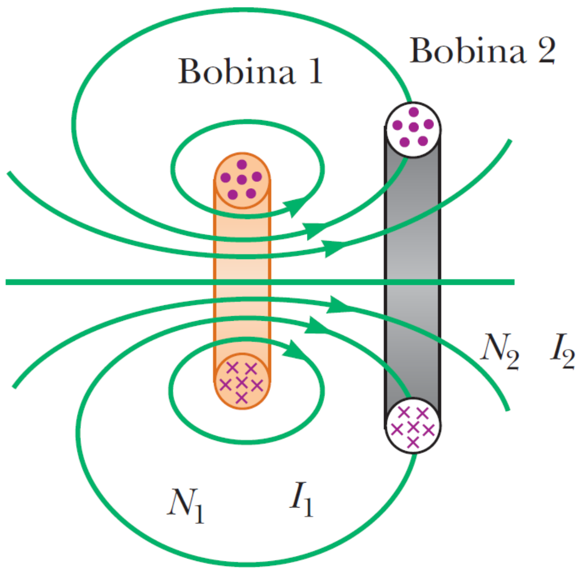
ϕ_{21} = flujo que atraviesa una vuelta de la bobina 2, debido al campo de la bobina 1

$$\phi_{21}^{total} = N_2\ \phi_{21}\ \rightarrow Proporcional\ a\ I_1$$

$$\phi_{21}^{total} = M_{21}\ I_1$$

M_{21}= Coeficiente de induccion mutua

Inducción mutua



$$\mathcal{E}_{21} = -\frac{d\phi_{21}^{total}}{dt} = -\frac{d(M_{21}\, I_1)}{dt} = -M_{21}\frac{dI_1}{dt}$$

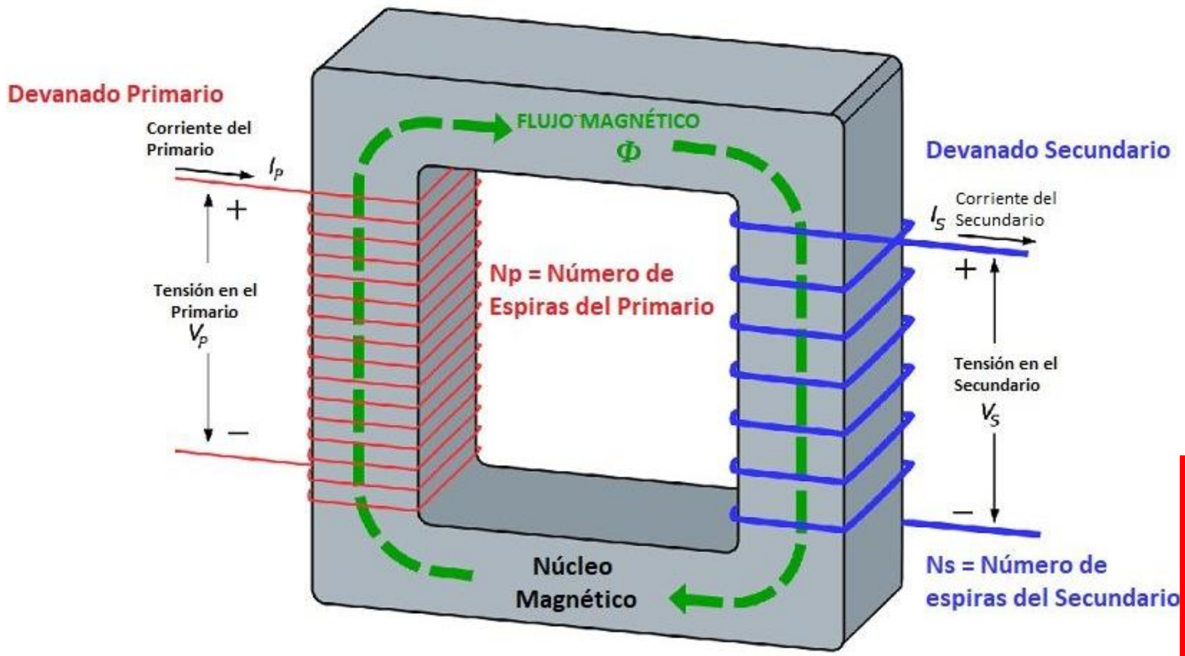
Si I_1 varia → Habrá fem inducida en la bob. 2
Si el circuito esta cerrado → Habrá $I_{inducida}$ en la bob. 2

$$\mathcal{E}_{12} = -\frac{d\phi_{12}^{total}}{dt} = -\frac{d(M_{12}\, I_2)}{dt} = -M_{12}\frac{dI_2}{dt}$$

Si I_2 varia → Habrá fem inducida en la bob. 1

$$M_{12} = M_{21} = M$$ → Depende de las posiciones relativas y dimensiones de cada espira

Transformador



$$\Delta V_P = -N_P \frac{d\phi_B}{dt}$$

$$\Delta V_S = -N_S \frac{d\phi_B}{dt}$$

$$\frac{d\phi_B}{dt} = -\frac{\Delta V_P}{N_P} = -\frac{\Delta V_S}{N_S}$$

Voltaje de salida

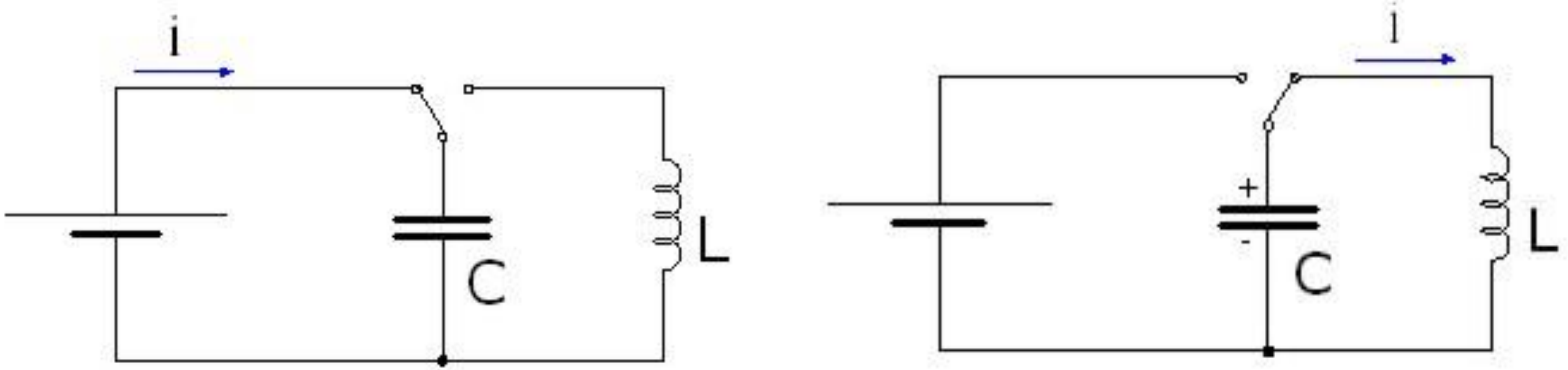
$$\Delta V_S = \frac{N_S}{N_P} \Delta V_P$$

Potencia

$$\Delta V_P \ I_P = \Delta V_S \ I_S$$

NUCLEO DE HIERRO: hace que el flujo sea igual en cada espira (concentra las líneas de B)

Circuitos LC – oscilaciones ideales

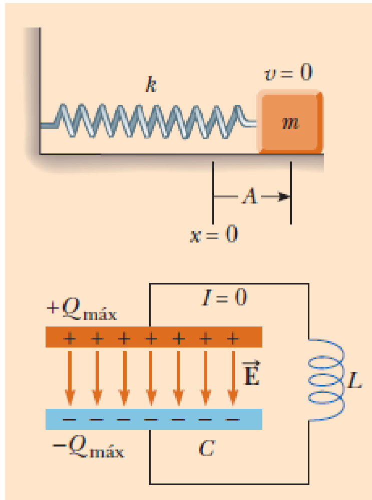


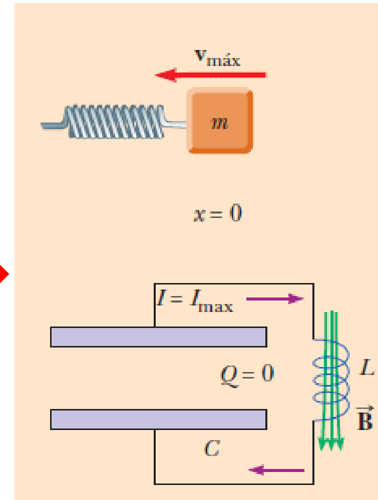
R es despreciable

Circuitos LC

$t = 0$

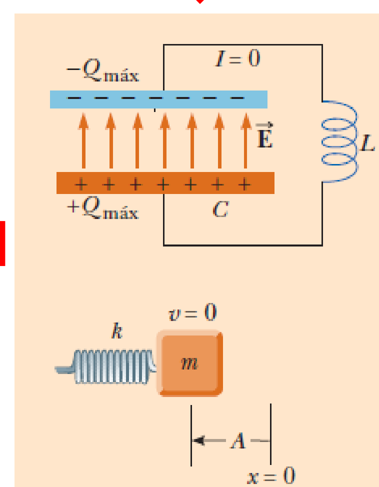
$$Q = Q_{max}$$
$$I = 0$$
$$U = U_C = \frac{Q_{max}^2}{2C}$$





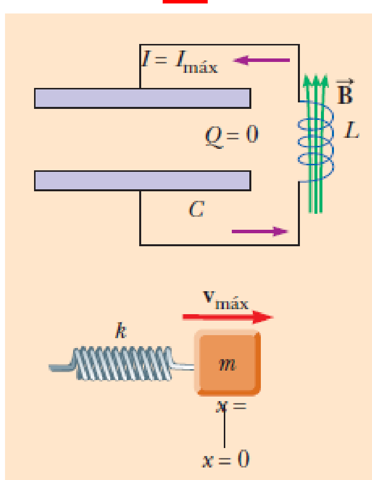
$t = \frac{1}{4}T$

$$Q = 0$$
$$I = I_{max}$$
$$U = U_L = \frac{L\,I^2}{2}$$



$t = \frac{1}{2}T$

$$Q = Q_{max}$$
$$I = 0$$
$$U = U_C = \frac{Q_{max}^2}{2C}$$



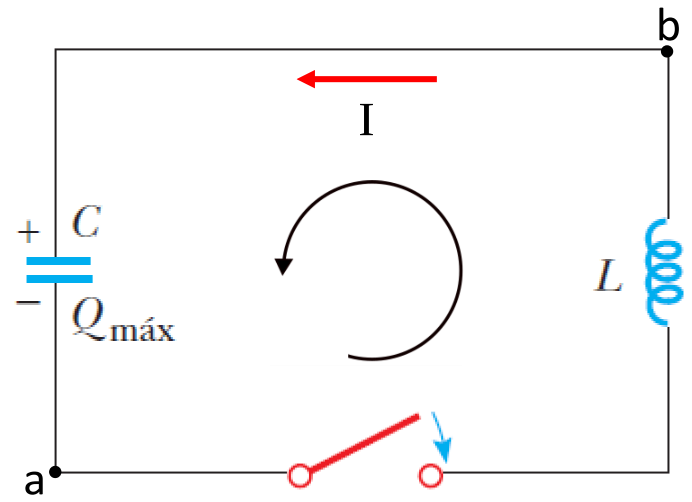
$t = \frac{3}{2}T$

$$Q = 0$$
$$I = I_{max}$$
$$U = U_L = \frac{L\,I^2}{2}$$

Circuitos LC – Frecuencia

$$(V_a - V_b) + (V_b - V_a) = 0$$

$$-V_C - V_L = -\frac{q}{C} - L\frac{di}{dt} = 0$$



Según el sentido de la corriente del dibujo

$$i = \frac{dq}{dt} \Rightarrow \frac{di}{dt} = \frac{d^2q}{dt^2}$$

$$\boxed{\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0}$$

Ecuación diferencial a resolver (similar al oscilador armónico)

Oscilador armónico

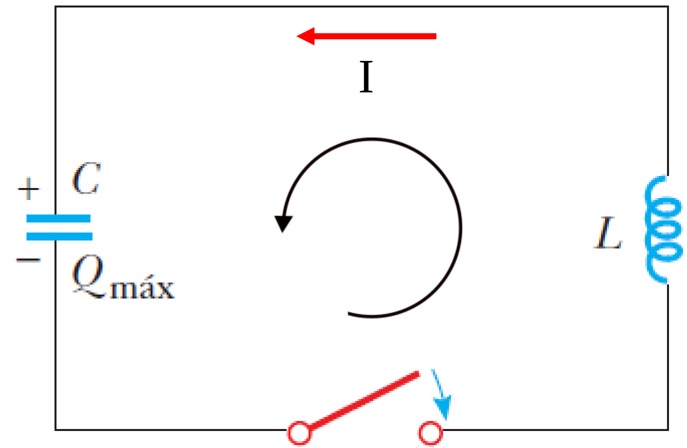
$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x \qquad x = A\cos(\omega t + \phi)$$

Circuitos LC – Frecuencia

$$q(t) = Q_M\, cos(\omega_0 t)$$

$$i(t) = \frac{dq}{dt} = \frac{d}{dt}[Q_M\, cos(\omega_0 t)\,]$$

$$i(t) = -Q_M \omega_0 sen(\omega_0 t) = -I_M sen(\omega_0 t)$$



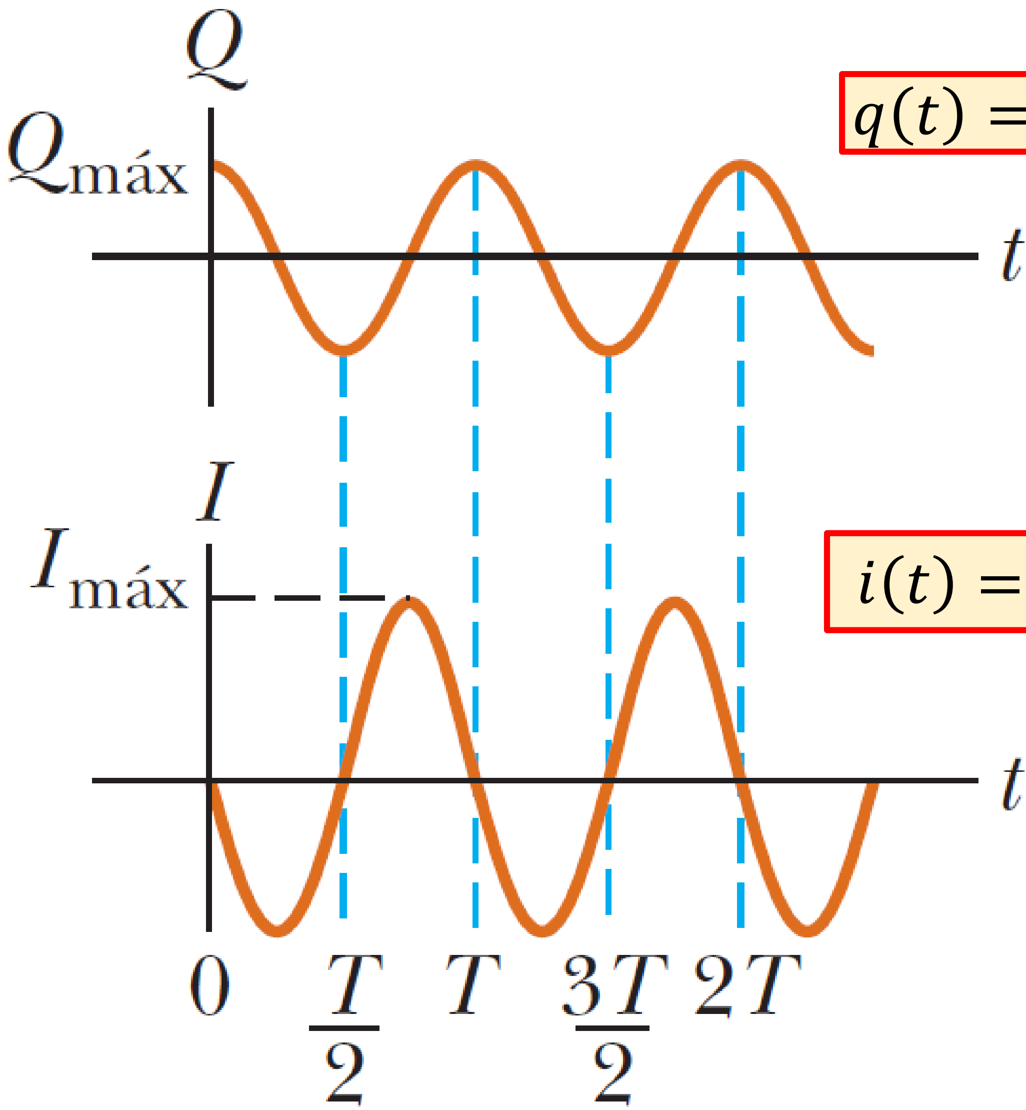
$$\frac{d^2 q}{dt^2} = -Q_M \omega_0^2 \cos(\omega_0 t)$$

$$-Q_M \omega_0^2 \cos(\omega_0 t) + \frac{1}{LC} Q_M\, cos(\omega_0 t) = 0$$

$$Q_M \omega_0^2 \cos(\omega_0 t) = \frac{1}{LC} Q_M\, cos(\omega_0 t) \longrightarrow \omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Circuitos LC – Carga y Corriente



$$q(t) = Q_M\, cos(\omega_0 t)$$

$$i(t) = -I_M sen(\omega_0 t)$$

Periodo de la oscilación

$$T = \frac{2\pi}{\omega_0}$$

Circuitos LC – Energía

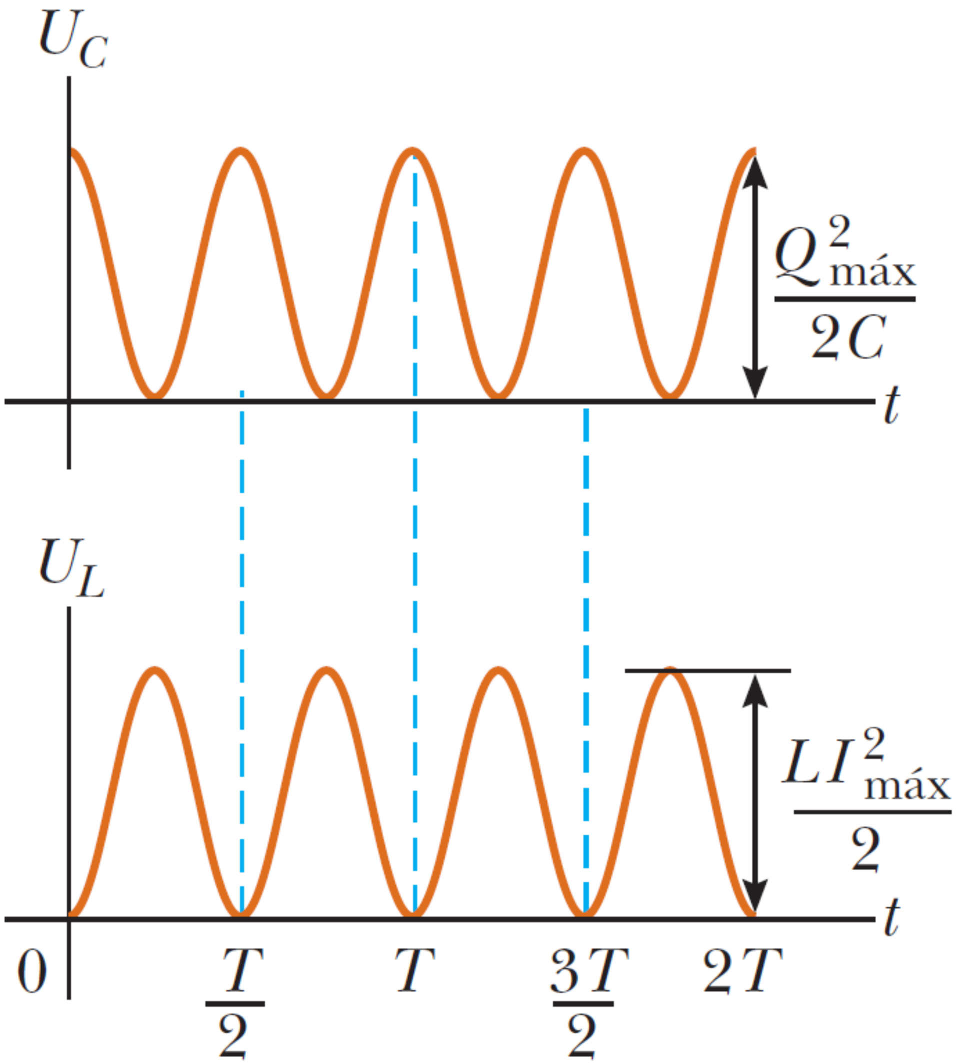
Como la energía no se transforma en energía interna (R=0) y no se transfiere al exterior $\Rightarrow$ $U = U_C + U_L = cte$

$$U = \frac{q(t)^2}{2C} + \frac{Li(t)^2}{2} = \frac{Q_M^2}{2C} cos^2\,(\omega_0 t) + \frac{LI_M^2}{2} sen^2(\omega_0 t)$$

Cuando U_C es máximo, U_L es cero y viceversa

$El\ valor\ cte\ de\ U\ a\ cualquier\ t\ es:$ $U = \frac{Q_M^2}{2C} = \frac{L\,I_M^2}{2}$

Circuitos LC – Energía



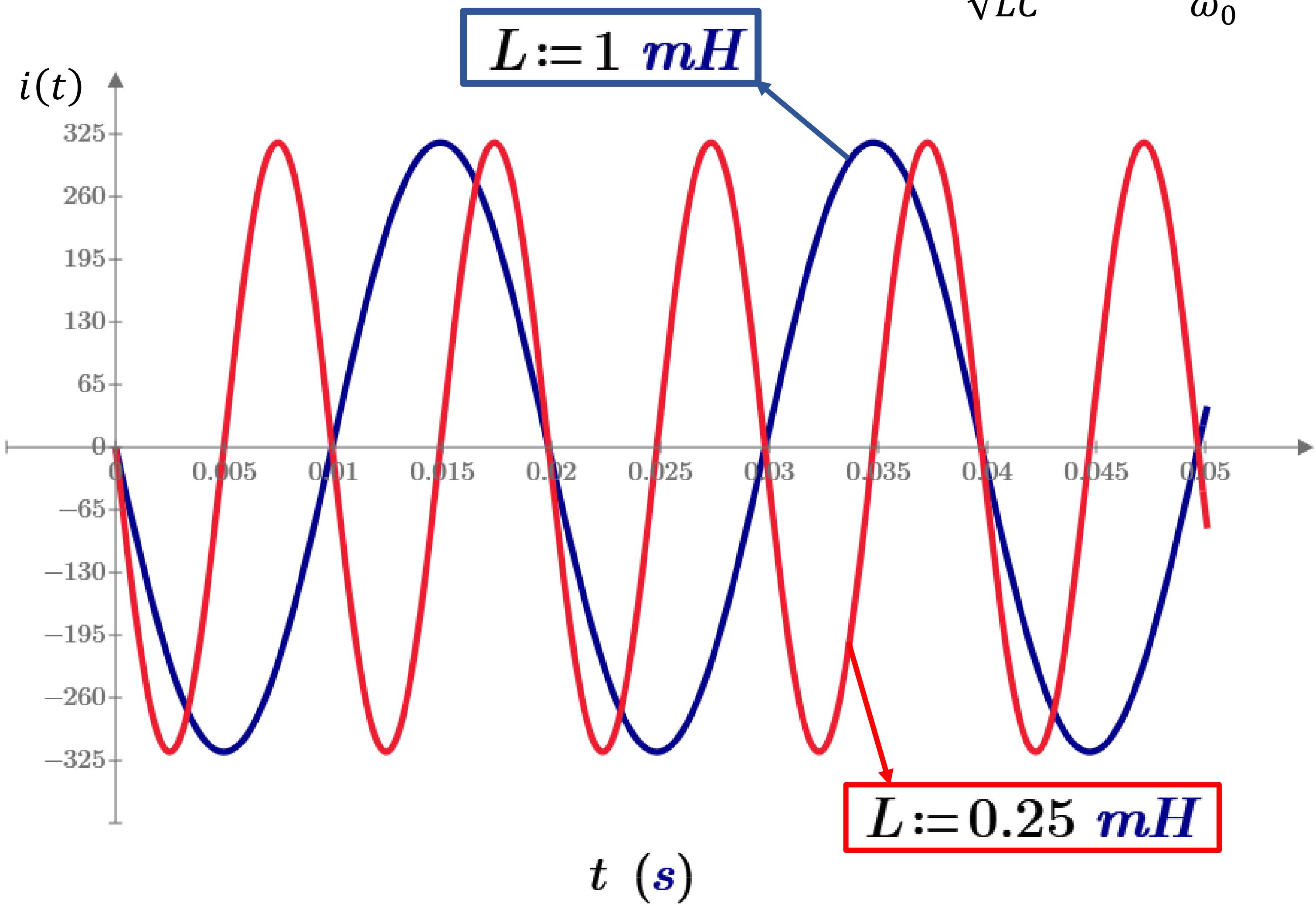
$$U_c = \frac{Q_M^2}{2C} cos^2\ (\omega_0 t)$$

$$U_L = \frac{1}{2} L I_M^2 sen^2 (\omega_0 t)$$

Circuito LC – Ejemplo

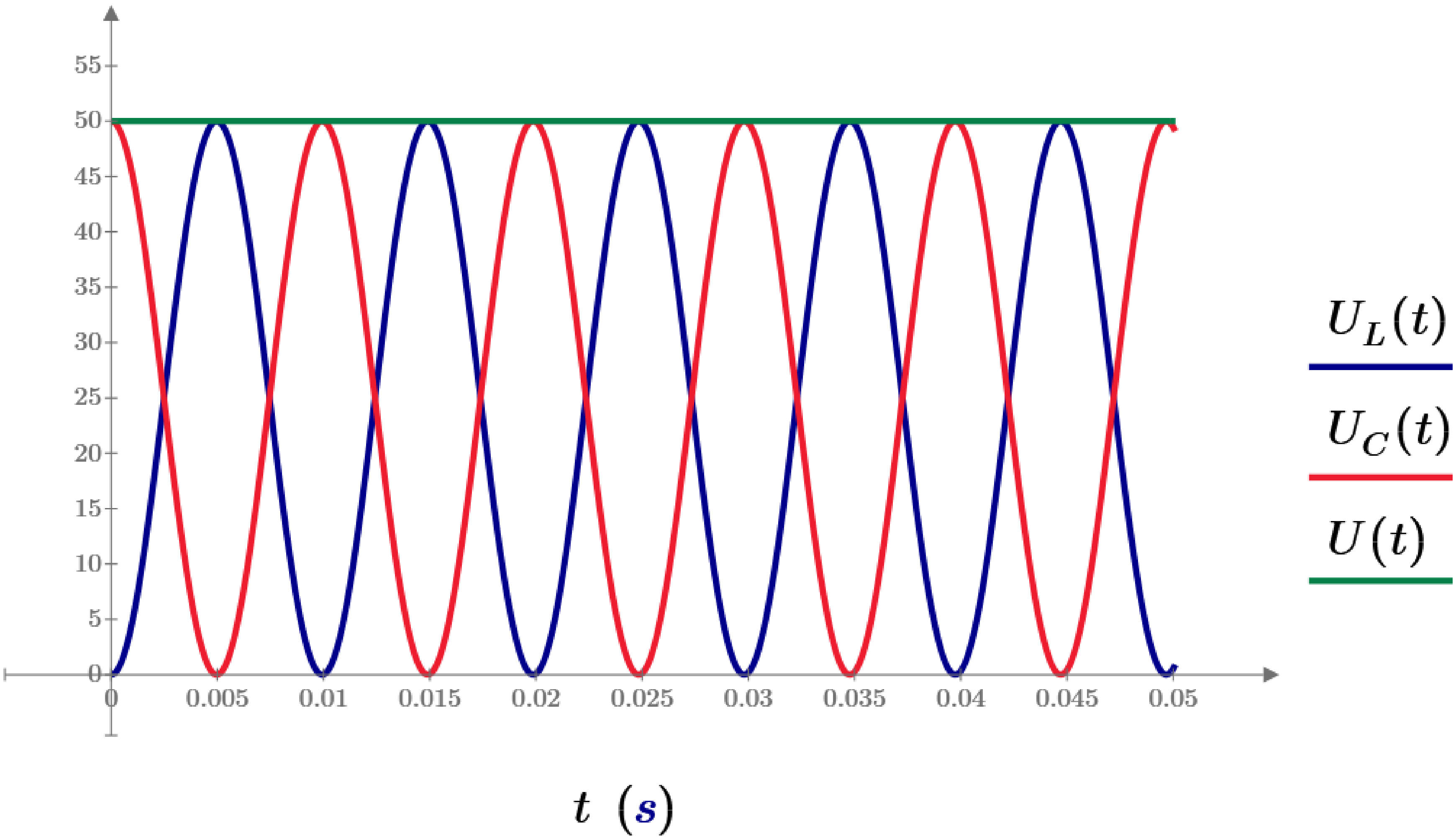
$C := 0.01\ F$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{LC}$$

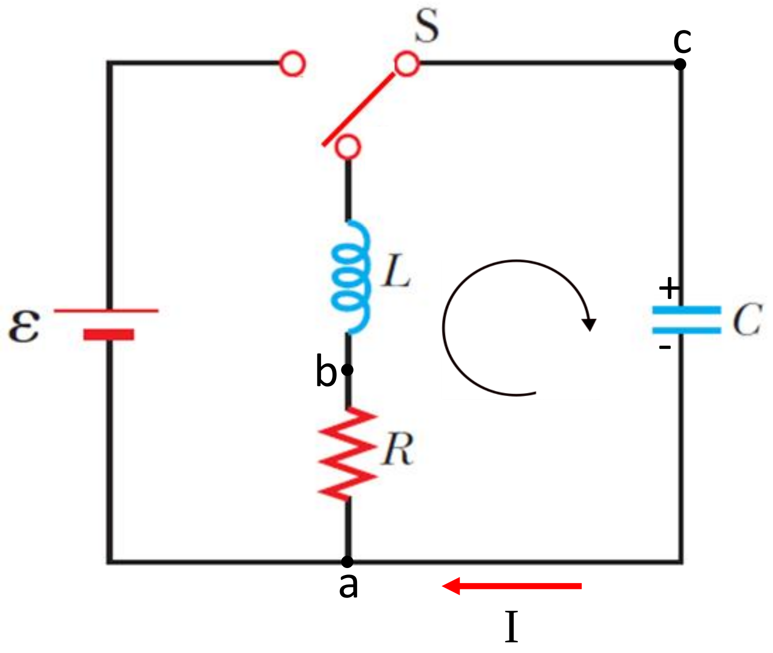


Circuito LC – Ejemplo

$C := 0.01\ F \qquad L := 1\ mH$



Circuitos RLC - oscilación amortiguada



$$(V_b - V_a) + (V_c - V_b) + (V_a - V_c) = 0$$

$$-V_R - V_L - V_C = -iR - L\frac{di}{dt} - \frac{q}{C} = 0$$

Ecuación diferencial a resolver

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = 0$$

Solución:

$$q(t) = Q_0\, e^{-\frac{Rt}{2L}} \cos(\omega' t)$$

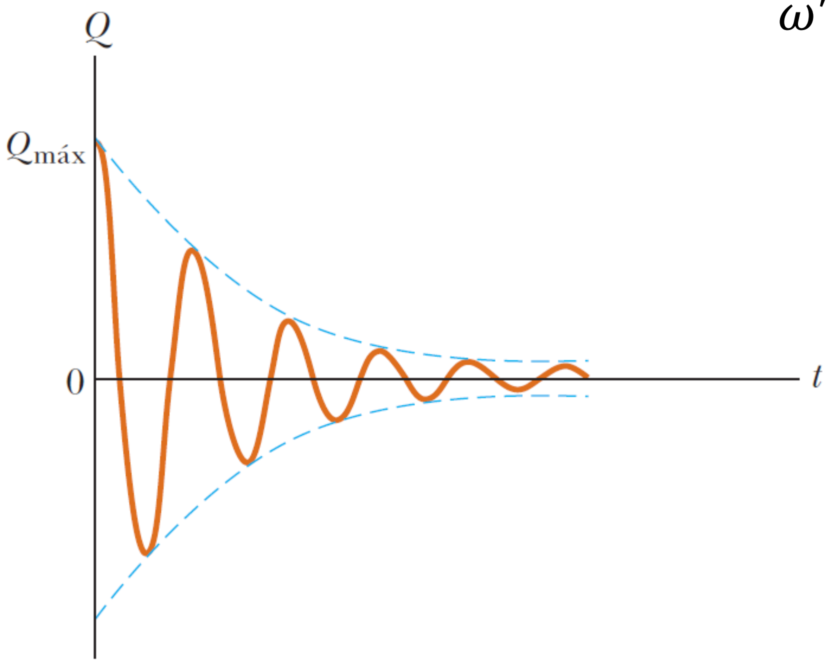
$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

Circuitos RLC - oscilación amortiguada

$$\tau = \frac{2L}{R}$$

$$q(t) = Q_0\, e^{-\frac{t}{\tau}} \cos(\omega' t)$$

$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{1}{\tau}\right)^2} = \sqrt{\omega_0^2 - \left(\frac{1}{\tau}\right)^2}$$



Si $R = 0$ → Circuito LC

Si $R \ll \sqrt{\frac{4L}{C}}$ (pequeña) → Asc. Amortiguada

Si $R \gg \sqrt{\frac{4L}{C}}$ (grande) → Sobreamortiguado

Si $R = \sqrt{\frac{4L}{C}}$ → Críticamente amort.

Circuitos RLC - Ejemplo

$$L := 100\ mH$$

$$C := 0.001\ F$$

$$\sqrt{4 \cdot \frac{L}{C}} = 20\ \Omega$$

$$q(t) = Q_0\, e^{-\frac{t}{\tau}} \cos(\omega' t)$$